Diagnosability Analysis

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Summary

The basic question to answer here is, which level of detectability and isolability is possible to achieve based on a given process model. This question is answered *without* designing a diagnosis system, it is a pure model property.

Model Form

The model is assumed to be in the form of a set of differential-algebraic equations where different subsets of equations are valid for different behavioral modes.

Example. A model of a battery with two modes OK and Deg:

Equations		Structure v	R	i	T
$v = u_0 - R \cdot i$	(1)	X	X	X	
$\dot{T} = \alpha(T_{\text{amb}} - T) + \beta i^2 R$	(2)		X	X	X
$\text{OK}(B) \rightarrow R = R_{\text{nom}}$	(3)	X			
$\text{Deg}(B) \rightarrow \dot{R} = 0$	(4)	X			

B is the mode of the battery; v, R, i, and T internal unknown variables; u_0, R_{nom}, and T_{amb} known constants. The equations (1)-(3) describes the behavior in the no-fault mode.

Generally, let the set of equations implied by mode b_i be denoted by

$$M_{b_i}(\dot{x}, x, z) = 0$$

where x are internal unknown variables, z known variables. Mode b_i is either the no-fault mode $b_0 = \text{NF}$ or a fault $b_i = f_i$.

Basic Definitions

Definition 1 (Observation set).

$$\mathcal{O}(b_i) = \{z | \exists x. M_{b_i}(\dot{x}, x, z, 0) = 0\}$$

The observation set is the set of all possible observations in a particular mode. Based on this, detectability and isolability are defined as:

Definition 2 (Detectability). A fault f_i is said to be detectable if

$$\mathcal{O}(f) \not\subseteq \mathcal{O}(\text{NF})$$

Definition 3 (Isolability). A fault f_i is said to be isolable from fault f_j if

$$\mathcal{O}(f_i) \not\subseteq \mathcal{O}(f_j)$$

Thus, a fault is *detectable* if there exists *any* fault situation that produces observations not consistent with the nominal model and correspondingly for the isolability requirement.

Diagnosability - Analytic

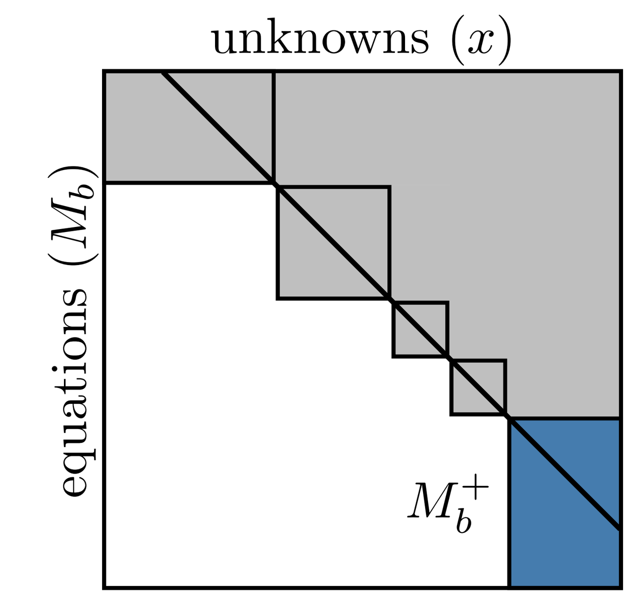
For linear DAEs, the observation sets can explicitly be expressed only in z by straightforward elimination of x.

Theorem 1. Let $\mathcal{O}(b_i) = \{z | R_i(p)z = 0\}$, then mode b_1 is not isolable from mode b_2 if and only if

$$\forall s \in \mathbb{C}.\ \text{rank} \begin{pmatrix} R_1(s) \\ R_2(s) \end{pmatrix} = \text{rank}\ R_1(s)$$

Diagnosability - Structural

For non-linear systems, structural methods can be used.

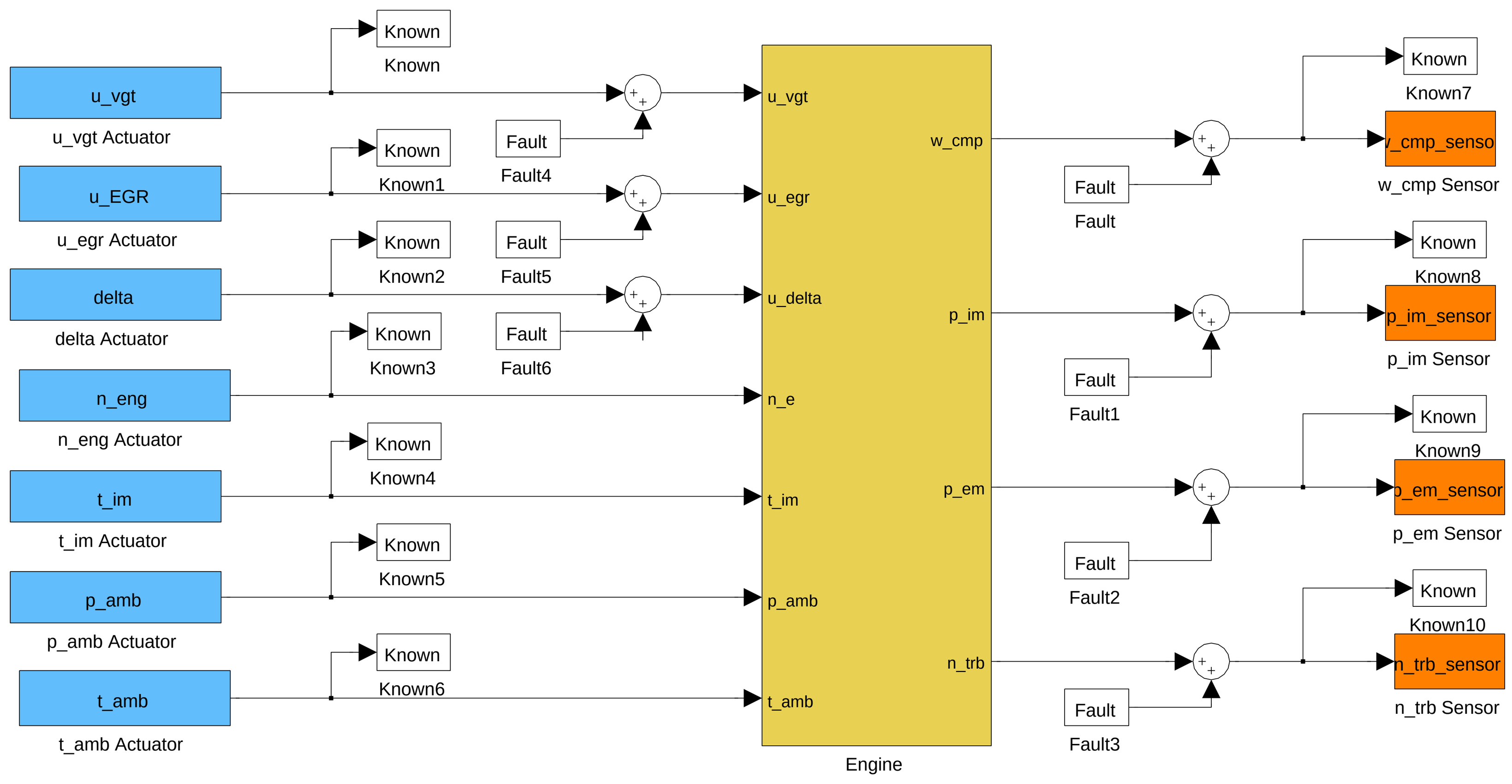


The figure shows the Dulmage-Mendelsohn's (DM) decomposition of the structure of M_b. The blue area indicates the part of the model that contains redundancy, i.e., the equations in M_b that can be tested.

Theorem 2. b_1 is not isolable from b_2 if $M_{b_2}^+ \subseteq M_{b_1}^+$

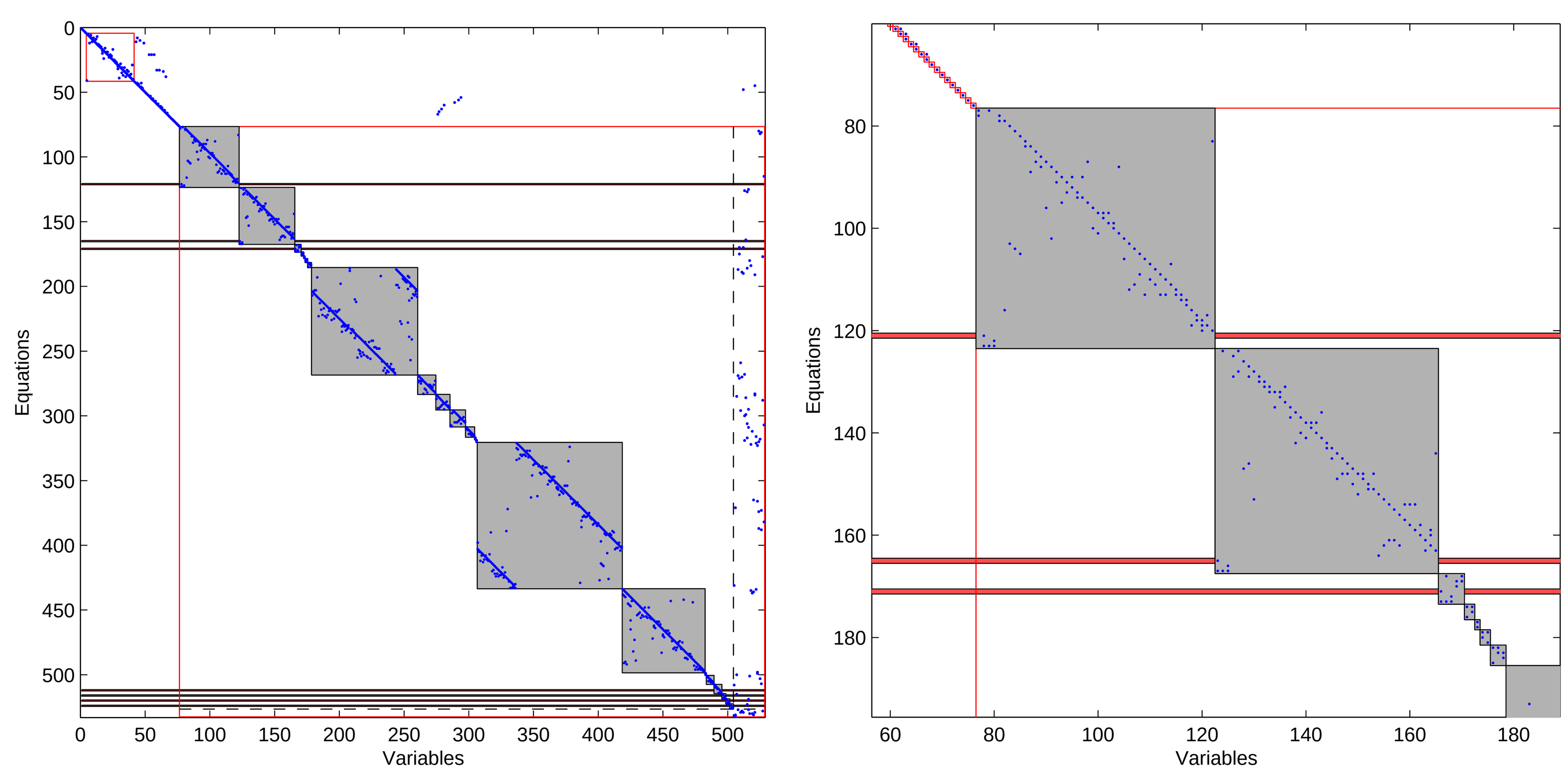
Intuition: $M_{b_2}^+ \subseteq M_{b_1}^+ \Rightarrow O(b_1) \subseteq O(b_2)$

Example - Diesel Engine Model



Model properties: 528 variables (8 states), 532 equations, 11 known variables, 7 faults.

Results: Structural analysis provided the following results in 0.17 seconds on a standard PC:



- The horizontal lines indicate equations related to faults.
- The red rectangles are blocks in the DM-decomposition.
- Faults affecting the same grey block are not isolable.

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